INFO216:
Knowledge Graphs

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Session S14: OWL DL

- Themes:
  - description logic
  - decision problems
  - OWL DL
  - Manchester OWL-syntax

Readings

- Forum links (cursory):
  - http://www.w3.org/TR/owl2-primer/
    - show: Turtle and Manchester syntax
    - hide: other syntaxes
  - Description Logic Handbook:
    - Chapter 1: Nardi & Brachman:
      Introduction to Description Logics
    - Chapter 2: Baader & Nutt:
      Formal Description Logics *(gets hard)*

Description Logic (DL)

Description logics

- Description Logic (DL)
  - a simple *fragment* of predicate logic
    - ...or, rather, a *family of such fragments*
  - not very *expressive* (“uttrykkskraftig”)
  - but (can have) *good decision problems*, i.e.,
    - it answers *decision problems* (rather) quickly
- Suitable for describing *concepts* (“begreper”)
  - formal basis for *OWL DL*
  - can be used to:
    - describe *concepts* and their *roles* (“Tbox”)
    - describe *individuals* and their *roles* (“ABox”)

Relationship to other logics

- *Proposition logics* are about *statements* (*propositions*):

  “Martha is a Woman” ⇐

  “Martha is Human” ∧ “Martha is Female”

- (First order) *predicate logics* are about *predicates* and *objects*:
  - **∀x.(Woman(x) ⇔ Human(x) ∧ Female(x))**
- *Description logics* are about *concepts*:
  - **Woman ≐ Human ⊓ Female**
  - ...and also about *roles* and *individuals*
- There are many other logic systems:
  - *modal logics*: necessarily □, possibly ◊
  - *temporal logics*: always □, sometimes ◊, next time ○

Definition of concepts (“begreper”)

- **Woman** ≐ **Human** ⊓ **Female**
- **Man** ≐ **Human** ⊓ ¬ **Woman**
- **Parent** ≐ **Mother** ⊔ **Father**
  - concepts: **Human, Female, Woman…**
  - definition: ≐
  - conjuction (and): ⊓
  - disjunction (or): ⊔
  - negation (not): ¬
  - nested expressions: **( )**
- **Childless** ≐ ..using Human and Parent..

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  - nested expressions: **( )**
- **Childless** ≐ **Human** ⊓ ¬ **Parent**

Types of concepts (“begreper”)

- **Woman ≐ Human ⊓ Female**
- **Man ≐ Human ⊓ ¬ Woman**
- **Parent ≐ Mother ⊔ Father**
  - atomic concepts: **Human, Female, Woman…**
  - complex concepts / concept expressions: **¬ Woman, Human ⊓ Female...**
  - (atomic) base concepts: **Human, Female…**
    - only used on the r.h.s. of definitions
  - (atomic) defined concepts: **Woman, Man…**
    - used on the l.h.s. of *a single* definition
    - *unequivocality*: each defined (or named) concept occurs in the l.h.s. of only one definition
    - acyclicity

l.h.s. = left-hand side, r.h.s. = right-hand side

Base and defined concepts and roles

- *Atomic base concepts* are given
  - *corresponds to OWL-NamedClasses that are not composed from other classes/properties/...*
- *Atomic defined / named concepts*
  - *corresponds to OWL-NamedClasses that are composed from other classes*
  - defined by *concept expressions*
  - name appears on the left side of an $\doteq$ definition
  - concept expression appears on the right side
- ...similar distinction between *base and defined roles* later

Roles

- **Mother** ≐ **Female** ⊓ **∃hasChild.**⊤
- **Bachelor** ≐ **Male** ⊓ **¬∃hasSpouse.**⊤
- **Uncle** ≐ **Male** ⊓ **∃hasSibling.Parent**
  - roles: **hasChild, hasSibling...**
  - universal concept (“top”): ⊤
  - existential restriction: **∃**
- **Grandparent** ≐ ..using Human, hasChild, Parent..
- **Grandparent** ≐ ..using only Human, hasChild..
- **Uncle** ≐ ..using Male, hasSibling, hasChild..

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- **Grandparent** ≐ **Human** ⊓ **∃ hasChild.∃ hasChild.**⊤
- **Uncle** ≐ ....using Male, hasSibling, hasChild....

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- **Grandparent** ≐ **Human** ⊓ **∃hasChild.Parent**
- **Grandparent** ≐ **Human** ⊓ **∃ hasChild.∃ hasChild.**⊤
- **Uncle** ≐ **Male** ⊓ **∃ hasSibling.∃ hasChild.**⊤

Null concept

- **Male ⊓ Female ⊑ ⊥**
  - null concept (“bottom”): ⊥
  - subsumption (sub concept): ⊑
  - equivalence: ≡
- ≐ is used for *definitions* (or just ≡)
- ≡ are used for *equivalence axioms*
- ⊑ are used for *subsumption axioms*
  - or: containment / specialisation axioms
- Note the use of . . . ⊑ ⊥ (“subsumption of bottom”)
  - to say that something is not the case
- *This was our first proper axiom!*
  - so far we have just defined *concepts*
  - we have not used them in proper *axioms*

Null concept

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  - null concept (“bottom”): ⊥
  - subsumption (sub concept): ⊑
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- ≐ is used for *definitions* (or just ≡)
- ≡ are used for *equivalence axioms*
- ⊑ are used for *subsumption axioms*
  - or: containment / specialisation axioms
- Note the use of . . . ⊑ ⊥ (“subsumption of bottom”)
  - to say that something is not the case
- *But:*
  - definitions are a special type of *equivalence axioms*
  - with a single atomic (defined) concept on the l.h.s.

More uses of roles

- **HappyFather** ≐ **Father** ⊓
  ∀ hasChild.HappyPerson
  - universal restriction: **∀**
- **MotherOfOne** ≐ **Mother** ⊓ **=1 hasChild.**⊤
- **Polygamist** ≐ **≥3 hasSpouse.**⊤
  - number restrictions: **=, ≥, ≤**
- **Narsissist** ≐ **∃hasLoveFor.Self**
  - **self references**: **Self**
- **MassMurderer** ≐ ...using hasKilled, Human...

More uses of roles

- **HappyFather ≐ Father ⊓ ∀hasChild.HappyPerson**
  - universal restriction: **∀**
- **MotherOfOne ≐ Mother ⊓ =1 hasChild.⊤**
- **Polygamist ≐ ≥3 hasSpouse.⊤**
  - number restrictions: **=, ≥, ≤**
- **Narsissist ≐ ∃hasLoveFor.Self**
  - **self references**: **Self**
- **MassMurderer ≐ ≥4 hasKilled.Human**

Inverse and transitive roles

- **Child** ≐ **Human** ⊓ ∃**hasChild**$^-$.⊤
- **hasParent** ≐ **hasChild**$^-$
- **hasSibling** ≐ **hasSibling**$^-$
- **BlueBlood** ≐ ∀**hasParent***.**BlueBlood**
  - inverse role: **hasChild**$^-$
  - symmetric role: **hasSibling**$^-$
  - transitive role: **hasParent***
- **Niece** ≐ ..Woman, hasChild, hasSibling..

Inverse and transitive roles

- **Child ≐ Human ⊓ ∃hasChild⁻.⊤**
- **hasParent ≐ hasChild⁻**
- **hasSibling ≐ hasSibling⁻**
- **BlueBlood ≐ ∀hasParent*.BlueBlood**
  - inverse role: **hasChild⁻**
  - symmetric role: **hasSibling⁻**
  - transitive role: **hasParent***
- **Niece ≐ Woman ⊓ ∃hasChild⁻.hasSibling.⊤**
- *We have started to define roles*
  - so far, we have only defined *concepts*

Composite roles

- Similar to composite concepts, e.g.:
  - **hasUncle ≐ hasParent o hasBrother**
  - **hasLovedChild ≐ hasChild ⊓ hasLoveFor**
  - **hasBrother ≐ (hasSibling | Male)**
- Mostly *not* supported by reasoning engines
  - they have “bad decision problems”
    - meaning that they compute slowly or intractably
  - ...with some exceptions
- **hasDaughter** ≐ ..using hasChild, Female..

Composite roles

- Similar to composite concepts, e.g.:
  - **hasUncle ≐ hasParent o hasBrother**
  - **hasLovedChild ≐ hasChild ⊓ hasLoveFor**
  - **hasBrother ≐ (hasSibling | Male)**
- Mostly *not* supported by reasoning engines
  - they have “bad decision problems”
    - meaning that they compute slowly or intractably
  - ...with some exceptions
- **hasDaughter ≐ (hasChild | Female)**

TBox

- *Terminology box* (TBox):
  - a collection of axioms and definitions
  - axioms are equivalences or subsumptions:
    - *equivalence axioms* (≡):
      - composite concept (role) expressions on both sides
    - *subsumption* axioms (⊑):
      - composite concept (role) expressions on both sides
  - terminology boxes can also contain definitions:
    - *definition axioms* (≐):
      - atomic defined / named concept (role) on the l.h.s.
      - composite concept (role) expression on the r.h.s
    - make it easier to write other axioms

Acyclic, definitional TBox

$$
\begin{aligned}
\text{Woman} &\equiv \text{Person} \sqcap \text{Female} \\
\text{Man} &\equiv \text{Person} \sqcap \neg\text{Woman} \\
\text{Mother} &\equiv \text{Woman} \sqcap \exists\text{hasChild}.\text{Person} \\
\text{Father} &\equiv \text{Man} \sqcap \exists\text{hasChild}.\text{Person} \\
\text{Parent} &\equiv \text{Father} \sqcup \text{Mother} \\
\text{Grandmother} &\equiv \text{Mother} \sqcap \exists\text{hasChild}.\text{Parent} \\
\text{MotherWithManyChildren} &\equiv \text{Mother} \sqcap \geqslant 3\,\text{hasChild} \\
\text{MotherWithoutDaughter} &\equiv \text{Mother} \sqcap \forall\text{hasChild}.\neg\text{Woman} \\
\text{Wife} &\equiv \text{Woman} \sqcap \exists\text{hasHusband}.\text{Man}
\end{aligned}
$$

Acyclic, and
contains only definitions!

TBox

- *Acyclic TBoxes:*
  - contains only definitions
  - subsumption axioms can (sometimes) be removed:
    - $T \sqsubseteq C$ is transformed into $T \doteq \overline{T} \sqcap C$
      - Example:

        Male $\sqsubseteq$ **Human** is transformed into

        Male $\doteq$ **Maleness** $\sqcap$ **Human**
    - *when only a single l.h.s. term*
  - every defined concept (or role) can be *expanded* into an expression of only atomic base concepts (or roles)
- *Expanded concepts* (or *roles*)
  - defined only in terms of *atomic base concepts* (and *roles*)
  - expanded, definitional TBox

Expanded definitional TBox

Only atomic base concepts on the right hand sides!

$$
\begin{aligned}
\mathsf{Woman} &\equiv \mathsf{Person} \sqcap \mathsf{Female} \\
\mathsf{Man} &\equiv \mathsf{Person} \sqcap \neg(\mathsf{Person} \sqcap \mathsf{Female}) \\
\mathsf{Mother} &\equiv (\mathsf{Person} \sqcap \mathsf{Female}) \sqcap \exists \mathsf{hasChild}.\mathsf{Person} \\
\mathsf{Father} &\equiv (\mathsf{Person} \sqcap \neg(\mathsf{Person} \sqcap \mathsf{Female})) \sqcap \exists \mathsf{hasChild}.\mathsf{Person} \\
\mathsf{Parent} &\equiv ((\mathsf{Person} \sqcap \neg(\mathsf{Person} \sqcap \mathsf{Female})) \sqcap \exists \mathsf{hasChild}.\mathsf{Person}) \\
&\quad \sqcup ((\mathsf{Person} \sqcap \mathsf{Female}) \sqcap \exists \mathsf{hasChild}.\mathsf{Person}) \\
\mathsf{Grandmother} &\equiv ((\mathsf{Person} \sqcap \mathsf{Female}) \sqcap \exists \mathsf{hasChild}.\mathsf{Person}) \\
&\quad \sqcap \exists \mathsf{hasChild}.(((\mathsf{Person} \sqcap \neg(\mathsf{Person} \sqcap \mathsf{Female})) \\
&\qquad \sqcap \exists \mathsf{hasChild}.\mathsf{Person}) \\
&\qquad \sqcup ((\mathsf{Person} \sqcap \mathsf{Female}) \\
&\qquad\quad \sqcap \exists \mathsf{hasChild}.\mathsf{Person})) \\
\mathsf{MotherWithManyChildren} &\equiv ((\mathsf{Person} \sqcap \mathsf{Female}) \sqcap \exists \mathsf{hasChild}.\mathsf{Person}) \sqcap \geqslant 3\, \mathsf{hasChild} \\
\mathsf{MotherWithoutDaughter} &\equiv ((\mathsf{Person} \sqcap \mathsf{Female}) \sqcap \exists \mathsf{hasChild}.\mathsf{Person}) \\
&\quad \sqcap \forall \mathsf{hasChild}.(\neg(\mathsf{Person} \sqcap \mathsf{Female})) \\
\mathsf{Wife} &\equiv (\mathsf{Person} \sqcap \mathsf{Female}) \\
&\quad \sqcap \exists \mathsf{hasHusband}.(\mathsf{Person} \sqcap \neg(\mathsf{Person} \sqcap \mathsf{Female}))
\end{aligned}
$$

Statements about individuals

- So far axioms about concepts and roles (*TBox*)
- Also two types of axioms about individuals (*ABox*):
  - *class assertion* (using a *concept*):

    Märtha : Female ⊓ Royal
  - *role assertion* (using a *role*):

    <Märtha, EmmaTallulah> : hasChild

    <Märtha, HaakonMagnus> : hasBrother
- *Axioms* about concepts/roles and *assertion axioms* about individuals/roles are used to create knowledge bases:
  - concepts, roles in the *TBox* (aka “the tags”)
  - individuals, roles in the *ABox* (“the tagged data”)

Syntaxes differ a bit...

- So far axioms about concepts and roles (*TBox*)
- Also two types of axioms about individuals (*ABox*):
  - *class assertion* (using a *concept*):

    Female(Märtha),(Female ⊓ Royal)(Märtha)
  - *role assertion* (using a *role*):

    hasChild(Märtha, EmmaTallulah)
    hasBrother(Märtha, HaakonMagnus)
- *Axioms* about concepts/roles and *assertion axioms* about individuals/roles are used to create knowledge bases:
  - concepts, roles in the *TBox* (aka “the tags”)
  - individuals, roles in the *ABox* (“the tagged data”)

Summary of axioms

- Terminology axioms (in the TBox):
  - subsumptions: $C \sqsubseteq D$
  - equivalences: $C \equiv D$
    - corresponds to: $C \sqsubseteq D,\ D \sqsubseteq C$
  - definitions: $A \doteq C$

C and D are *expressions*, A is a *defined concept*!

- Individual assertion axioms (in the ABox):
  - class assertions: $a:C$
  - role assertions: $\langle a,b \rangle : R$

a and b are *individuals*. R is a *role*!

- A knowledge base $\mathbf{K = (\ T, A\ )}$ consists of
  - TBox: **T** and ABox: **A**

Decision Problems

Reasoning over knowledge bases

- *What more can we do with ontologies?*
- For example:
  - a *security ontology* that describes an organisation and its computer systems as concepts, roles and individuals
  - can answer *competency questions*, e.g.:
    - are all the *security levels* subclasses of one another?
    - what is the highest security level of a *temporary*?
    - what is the necessary security level of a *component*?
    - which employees have access to *critical data*?
    - for which *security roles* is an employee qualified?
    - which individuals are *suspicious persons*?
  - *DL offers a clear and compact way or representing and reasoning about questions such as these!*

Decision problems

- A computational problem with a yes/no answer, e.g.
  - is C *subsumed* by D ($\mathbf{K} \models \mathtt{C} \sqsubseteq \mathtt{D}$)?
  - are C and D *consistent* ($\mathbf{K} \models \mathtt{a:(C} \sqcap \mathtt{D)}$)?
  - does *a* *belong* to C ($\mathbf{K} \models \mathtt{a:C}$)?
  - is *a* *R-related* to *b* ($\mathbf{K} \models \mathtt{<a,b>:R}$)?

C and D are classes, a and b are individuals. R is a role!

- *Decidability ("bestembarhet")*:
  - we can always calculate the yes/no answer in finite time
- *Semi-decidability ("semibestembarhet")*:
  - we can always calculate a yes-answer in finite time,
    ...but not always a no-answer
- *Undecidability ("ubestembarhet"):*
  - we cannot always calculate the answer in finite time

Decision problems for concepts

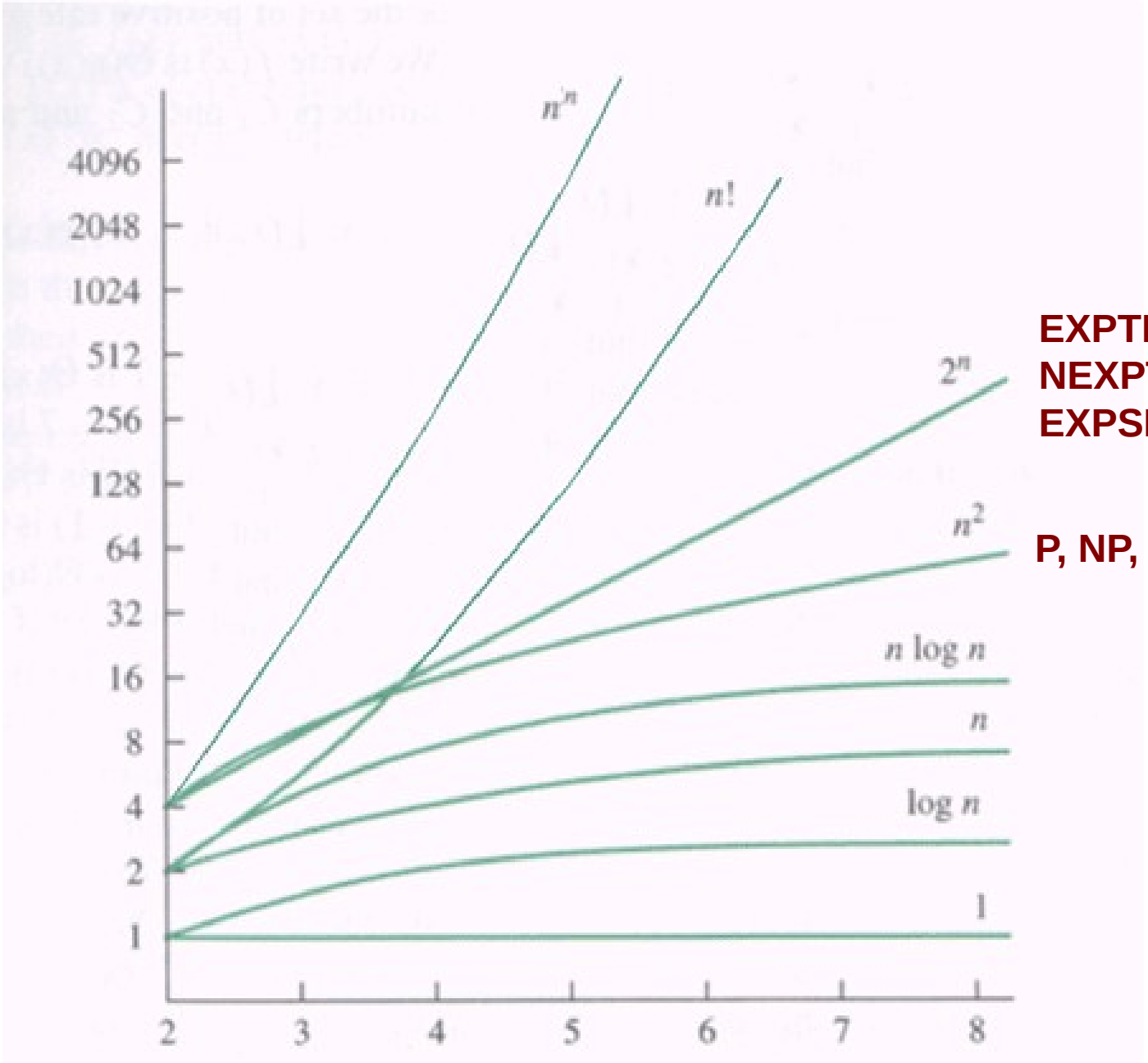
- There are four basic decision problems for concepts:
  - consistency: whether there is an individual $\mathbf{a}$ so that
    $\mathbf{T} \models \mathbf{a}:\mathbf{C}$,
    $\mathbf{T} \not\models \mathbf{C} \sqsubseteq \bot$
  - subsumption: $\mathbf{T} \models \mathbf{C} \sqsubseteq \mathbf{D}$,
    $\mathbf{T} \models \mathbf{C} \sqcap \neg\mathbf{D} \sqsubseteq \bot$
  - equivalence: $\mathbf{T} \models \mathbf{C} \equiv \mathbf{D}$ *or* $\mathbf{C} \equiv_{\mathbf{T}} \mathbf{D}$,
    $\mathbf{T} \models \mathbf{C} \sqsubseteq \mathbf{D}$, $\mathbf{D} \sqsubseteq \mathbf{C}$
  - disjunction: $\mathbf{T} \models \mathbf{C} \sqcap \mathbf{D} \sqsubseteq \bot$
- *All four can be reduced to subsumption or consistency!*
- $\mathbf{T}$ can be *emptied*, by expanding all its concepts

Decision problems for individuals

- Decision problems for individuals and roles:
  - instance checking: $\mathbf{A} \models \mathbf{a:C}$,
    $\not\models \mathbf{A} \sqcap \neg\mathbf{(a:C)}$
    *is individual **a** member of class/concept **C**?*
  - role checking: $\mathbf{A} \models \mathbf{\langle a,b\rangle:R}$,
    $\not\models \mathbf{A} \sqcap \neg\mathbf{(\langle a,b\rangle:R)}$
    *is individual **a** **R**-related to individual **b**?*
  - classifications (not yes/no):
    to which classes/concepts does **a** belong?
    all individuals of class/concept **C**?
- *Everything boils down to consistency checking for ABoxes*
  - ...under certain (rather weak) conditions

Complexity

- Decidability is often necessary
  - but not enough
  - we also want a decision “in reasonable time”
  - different DL-variants have different *complexity*
  - many different *complexity classes*
    - polynomial (**P**), exponential (**EXP**)...
    - ...in time and space
- *Tractable* (or *feasible*) complexity
  - acceptable complexity for large knowledge bases
  - typically *polynomial* complexity (**P**)
  - complexity grows $O(n^c)$ of problem size n



EXPTIME, NEXPTIME, EXPSPACE

P, NP, PSPACE

DL-complexity

- We have presented many DL-notations
  - *do not* use all at the same time!
  - that gives high complexity
  - which is why we have different OWL Profiles
- Complexity calculator on the net:
  - *Complexity of reasoning in Description Logics*
    http://www.cs.man.ac.uk/~ezolin/dl/

OWL DL

Relation to OWL

- OWL DL and description logic are closely matched
  - everything in OWL DL has a DL-counterpart
  - most everything in DL can be expressed in OWL DL
- DL is a family of logic systems:
  - some of them correspond to particular OWL profiles
  - OWL1 DL: $\mathcal{SHOIN}(\mathbf{D})$
  - OWL2 DL: $\mathcal{SROIQ}(\mathbf{D})$

OWL profiles revisited

- OWL "1" (2002):
  - *OWL Full – "anything goes"*
  - OWL DL – fragment of OWL Full,
    - formal semantics through *description logic*
  - OWL Lite – simple fragment of OWL DL, not much used
- OWL 2 (2008):
  - *OWL2 Full – "anything goes"*
  - OWL2 DL – fragment of OWL2 full, extension of OWL DL
    - OWL2 EL – quick reasoning, fragment of OWL2 DL
    - OWL2 RL – rule language, fragment of OWL2 DL
      - OWL LD – linked data, fragment of OWL2 RL
    - OWL2 QL – query language, fragment of OWL2 DL

And there is more...

- A few other constructions
- Formal definitions of
  - syntax (rules for valid expressions, reasoning)
  - semantics (rules for interpreting expressions)
- Tools and techniques
- Lots of applications

Protege-OWL

Protege-OWL

- Extension of Protegé
  - ordinary Protegé supports *frames*
  - Protegé-OWL
    - reuses much of the Protege-Frames GUI

Old Protege-OWL (3.x and older)

- Supported OWL 1.1:
  - used *Jena* internally
  - wrapped Jena's API with a *Protege-OWL API*
    - uses Jena's graph metaphor
    - you “create the ontology as a graph”
  - many plug-ins:
    - SWRL, Jess, reasoning...
  - still available,
    - but not so actively developed

Protege-OWL 4 and later

- Supports OWL 2:
  - complete reimplementation of internals
  - *not* based on Jena
  - offers a dedicated *OWL API* (in Java)
    - description-logic metaphor
    - you “build the ontology from axioms”
  - more and more plug-ins
  - most OWL DL reasoners have moved to the OWL API

Manchester OWL syntax

Manchester OWL-syntax

- A simple DL notation without special symbols
  - used by Protege-OWL to construct classes
  - similar to DL syntax
- ```
  Class: Woman
      EquivalentTo: Human and Female
  ```
- ```
  Class: Man
      EquivalentTo: Human and not Female
  ```
- ```
  Class: Parent
      EquivalentTo: Mother or Father
  ```
- Can be used to *serialise* complete ontologies
  - ...we will look mostly at TBox expressions
- http://www.w3.org/TR/owl2-manchester-syntax/
  ```

Comparison

- DL:

  Male ≐ Human ⊓ ¬Female

- Machester OWL:

```
Class: Man
    EquivalentTo: Human and not Female
```

- TURTLE:

```
family:Man owl:equivalentClass
    owl:intersectionOf (
        family:Human
        [   a owl:Class ;
            owl:complementOf family:Woman
        ]
    ) .
```

Roles in Manchester OWL syntax

- **Class: Mother**
  EquivalentTo:
  Female and hasChild some owl:Thing
- **Class: Bachelor**
  EquivalentTo:
  Male and not hasSpouse some owl:Thing
- **Class: Uncle**
  EquivalentTo:
  Male and hasSibling some Parent
  - universal concept (top): **owl:Thing**
  - existential restriction: **some**

Null concept in Manchester OWL syntax

- **Class:** <class-name>
  EquivalentTo: **Male** **and** **Female**
  SubClassOf: **owl:Nothing**
  - null concept (bottom): **owl:Nothing**
  - subsumption (subconcept): **SubClassOf:**
  - equivalence: **EquivalentTo:**
    - ...used both for *definitions* and for *axioms*

More roles in Manchester OWL syntax

- **Class:** HappyFather
  EquivalentTo:
  Father **and** hasChild **only** Happy
  - value restriction: **only**
- **Class:** MotherOfOne
  EquivalentTo: Mother **and**
  hasChild **exactly** 1
- **Class:** Bigamist
  EquivalentTo: hasSpouse **min** 2
  - number restriction: **exactly**, **min**, **max**
- **Class:** Narcissist
  EquivalentTo: loves **some** Self

Inverse, symmetric and transitive roles

- **Class: Child**
  EquivalentTo:
  Human and inverse hasChild some owl:Thing
- **Class: hasParent**
  EquivalentTo: inverse hasChild
- **ObjectProperty: hasSibling**
  Characteristic: Symmetric
- **ObjectProperty: hasAncestor**
  Characteristic: Transitive
- inverse role: **inverse**
  - symmetric role:
    Characteristic: SymmetricProperty
  - transitive role:
    Characteristic: TransitiveProperty